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OPTIMAL MANAGEMENT OF WIND POWER PRODUCTION VIA DISPATCHABLE ENERGY SOURCES

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Abstract

In this paper, a non-linear optimization problem for the optimal management of wind energy using dispatchable energy sources (DES) is solved. The model accounts for several aspects such as a penalty scheme in the wind power underproduction's case and the modeling of both electricity prices and wind energy production. All the quantities involved in the model are commented and computed through an empirical investigation based on a supposed wind farm.

Key words: Optimization problem, wind energy, copula function, electricity prices.

1. INTRODUCTION

Unlike many conventional energy sources, wind energy is highly unpredictable and uncertain as it depends on uncontrollable external factors like the weather. Its stochastic nature requires the wind energy producer (WPP) to manage promptly this uncertainty and, to this end, several solutions have been developed in the literature. One possibility for the WPP is to subscribe an insurance contract with a dispatch energy producer to protect from any energy underproduction (D'Amico, Petroni and Prattico, 2017). Another chance is constituted by the purchase of financial derivatives such as the call options. These financial products give the buyer the right to get the electricity at a fixed strike price instead of a spot price. Accordingly, the seller (the wind farm), receives in exchange a premium fee that is the call price (Bidwell, 2005). Finally, another possibility consists in the storage system based on the coordination of wind power generation with reserves in form of dispatchable energy sources like gas, which can be part of the energy portfolio of the WPP or can be bought on the market (Collet, F´eron and Tankov, 2017)-(Heredia, Cuadrado and Corchero, 2018)-(D'Amico, Petroni and Sobolewski, 2019)-(D'Amico, Di Basilio and Petroni, 2021).

This paper, which is nestled within this last research field, proposes a methodology to develop optimal coordination strategies of wind energy with dispatchable energy sources. In particular, it is an extension of the model already presented in the literature in which, by solving a nonlinear optimization problem, the optimal quantity of wind energy and DES to be produced was determined (D'Amico et al., 2021). With the improvement proposed in this work, it is possible to identify not only the optimal combination of wind energy and DES to be produced but also the optimal quantity of energy to be offered on the market. Firstly, it is assumed that electricity prices follow a Lognormal distribution and therefore that there are no negative prices. This is reasonable as negative prices are rare and are the result of temporary market imbalances (Fanone, Gamba and Prokopczuk, 2013). Secondly, it is supposed that wind power generation has a mixed discrete-continuous distribution. This hypothesis allows considering the effects of wind speeds lower than the cut-in speed and of wind speeds greater than the cut-off speed. Thirdly, it is considered a penalty in case of underproduction with respect to the quantity to deliver by contract. Specifically, the penalty is an increasing function of the energy not supplied (ENS) and it is described by a general power function of parameter. Finally, a Fairlie-Gumbel-Morgenstern (FGM) copula is used to shape the dependence between electricity prices and wind energy production. All these aspects are involved in the definition of the expected profit function which is maximized under a budget constrain in order to obtain both the optimal quantity of energy to be produced with DES and the optimal quantity of energy to deliver.

The rest of the paper is organized as follows: Section 2 displays the optimization problem and its solutions. Section 3 contains an empirical analysis of all the quantities involved in the proposed model. Section 4 presents conclusions and future goals.

2. OPTIMIZATION PROBLEM

Generally, electricity markets operation requires that at the present time ($t = 0$) the wind energy producer (WPP) proposes a certain amount of energy K to be placed on for the following period ($t = 1$). The main feature of these markets is the sanctioning system whose purpose is to punish operators providing a quantity of energy different than the promised one. This means that, if the WPP supplies a quantity of energy other than K, he will suffer a loss. Basically, the reason why there are discrepancies between what is offered at $t = 0$ and what is actually provided at $t = 1$ lies in the stochastic nature of the wind which makes wind energy production uncertain. To manage this risk and thus avoid penalties, at $t = 0$ the WPP purchases a certain amount of dispatchable energy source (DES).

The proposed methodology is an extension of that already presented in a previous paper in which, by solving a non-linear optimization problem, the optimal combinations of wind energy and DES to be produced for a given quantity K was determined (D'Amico et al., 2021). In the model below, in addition to determining the optimal coordination of wind energy and DES, it is also possible to detect the optimal quantity of energy K to be offered on the market at $t = 1$.

Hypotheses

H1: Let π_e be the electricity price at $t = 1$. At $t = 0$, this price is a positive and unknown random variable which is assumed to have a Lognormal distribution, ($\pi_e \sim$ Lognormal (μ, σ^2) with cumulative distribution function:

$$F_{\pi_e}(x) := \mathbb{P}[\pi_e \leq x] = \Phi\left(\frac{\ln(x) - \mu}{\sigma\sqrt{2}}\right),$$

where Φ is the cumulative distribution function of the standard normal distribution. The choice of the Lognormal distribution is motivated by computational reasons since it allows to obtain quasi-explicit solutions of the optimization problem. However, it should be remarked that in literature also other distributions, such as the Normal distribution (D'Amico et al., 2019), the Generalized Pareto distribution (Paraschiv, Hadzi-Mishev and Keles, 2016) or the Box-Cox

Power Exponential distribution (Bello, Bunn, Reneses and MuÜnoz, 2016), have been used to shape the electricity prices.

Let π_g be the cost of producing one unit of energy by DES at $t = 0$. It is a non-negative and known quantity.

H2: Let denote by *We* the quantity of energy produced using wind. It is a non-negative random variable since its value at $t = 1$ cannot be known at $t = 0$ because of many features (wind speed, wind direction, thermal stratification, and so on) which have random outcomes at $t = 1$. According to the previous literature (D'Amico et al., 2021), it is supposed that We has a mixed discrete-continuous distribution:

$$F_{we}(p) = \begin{cases} 0 & \text{if } p < 0 \\ a + (1-a)F(p) & \text{if } p \geq 0 \text{ and } a \in [0,1], \end{cases}$$

where $F(p)$ is an absolutely continuous cumulative distribution function (CDF) of a random variable W and a is a point mass at zero, *i.e.* $We \sim a\delta_0 + (1 - a)W$. The use of a mixed distribution permits regarding also the extreme cases in which the wind speed is too strong or too weak. In fact, a wind speed greater than the cut-off speed can damage the rotor of the blade and as a consequence, the blade is switched to a standstill state. Conversely, when the wind speed is lower than the cut-in speed, the blade is not able to rotate and generate power. It should be noted that the random variable We admits a probability density function:

$$f_{we}(p) = \frac{\partial F_{we}(p)}{\partial p} \text{ for all } p > 0. \tag{2.1}$$

Let denote by P_g the quantity of energy produced by DES which has to be optimally determined. It has to belog to closed set $[0; K]$.

Let introduce the energy not supplied (ENS) defined as

$$ENS := (K - (We + Pg))^+ = \max(0, K - (We + P_g)). \tag{2.2}$$

According to formula (2.2), the WPP offers K on the market and tries to do this by using both We and P_g quantities. If the total production $(We + P_g)$ doesn't achieve K, then there will be an energy not supplied (ENS) and the WPP will incur a cost of $\widetilde{C} \geq 0$ Euros where $\widetilde{C}$ is a function of ENS, *i.e.*:

$$\widetilde{C} = \mathrm{C} \cdot (ENS)^{\alpha} \text{ with } > 0, C \geq 0.$$

Finally, suppose that if there is an energy production that overcomes K, it is not sold at the market but it is lost.

H3: Let $F_{(\pi_e, we)}(x, p) = \mathbb{P}[\pi_e \leq x, We \leq p]$ be the joint cumulative distribution function of the wind power production and energy price at $t = 1$. Since these two random variables are not independent, a chance to model their joint distribution $F(\pi_e, we)(x; p)$ is given by a copula function (Durante and Sempi, 2016). According to the literature (D'Amico et al., 2021)-(D'Amico et al., 2019), a good choice could be considering a Fairlie-Gumbel-Morgenstern (FGM) copula applied to the marginal distributions $F_{we}(p)$ and $F_{\pi_e}(x)$:

$$F(we, \pi_e)(p, x) = C(F_{we}(p), F_{\pi_e}(x)).$$

The selection of the FGM copula answers the need to have quasi-explicit calculations of the optimal solutions. It is the only one that is a quadratic polynomial in u and v

$$C_\theta(u, v) = uv + \theta uv(1 - u)(1 - v), \quad \theta \in [-1, 1],$$

where θ is the dependence parameter. If $\theta = 0$, then the case of independence between the two random variables is recovered. From the definition of the FGM copula is obtained

$$F_{(we,\,\pi_e)}(p, x) = F_{we}(p)F_{\pi_e}(x)[1 + \theta(1 - F_{we}(p))(1 - F_{\pi_e}(x))],$$

and by differentiation for all $p > 0$ and for all $x \geq 0$

$$f_{(we,\,\pi_e)}(p, x) = f_{we}(p)f_{\pi_e}(x)[1 + \theta(1 - 2F_{we}(p))(1 - 2F_{\pi_e}(x))],$$

while for $p = 0$ and all $x \geq 0$ it follows that

$$\begin{aligned} f_{(we,\,\pi_e)}(0, x) &= \left[\frac{\partial C}{\partial v}(F_{we}(0), F_{\pi_e}(x)) - \frac{\partial C}{\partial v}(F_{we}(0^-), F_{\pi_e}(x))\right] f_{\pi e}(x) \\ &= [C_{we|\pi_e}(F_{we}(0)|F_{\pi_e}(x)) - C_{we|\pi_e}(F_{we}(0^-)|F_{\pi_e}(x))] f_{\pi e}(x) \\ &= \Delta C_{we|\pi_e}(F_{we}(0)|F_{\pi_e}(x)) f_{\pi e}(x) \\ &= \Delta C_{we|\pi_e}(a|F_{\pi_e}(x)) f_{\pi e}(x). \end{aligned}$$

To define the optimization problem, it necessary to introduce the profit function as the random variable $\mathcal{R}$ defined by

$$\mathcal{R} := \mathcal{R}_{>} + \mathcal{R}_{=},$$

where

$$\begin{aligned} \mathcal{R}_{>} := &\ \chi\{We + P_g \geq K\}\chi\{We > 0\}(\pi_e K - \pi_g P_g) \\ &+ \chi We + P_g < K\}\chi\{We > 0\}[\pi_e(We + P_g) - \pi_g P_g \\ &- C(K - (We + P_g))^{\alpha}], \end{aligned} \tag{2.3}$$

and

$$\mathcal{R}_{=} := \chi\{We = 0\}[\pi_e P_g - \pi_g P_g - C(K - P_g)^{\alpha}], \tag{2.4}$$

where $\chi(A)$ is the indicator function of event A. Relation (2.3) states that if the total energy produced ($W_e + P_g$) is greater than K, then the WPP will have a revenue from the energy sale of $\pi_e K$ and a cost, deriving from the purchase of P_g, equal to $\pi_g P_g$. Conversely, if the total energy produced is less than K, the WPP has an inflow $\pi_e(We + P_g)$, a cost generated by the acquisition of energy produced by DES of $\pi_g P_g$, and a further loss due to the penalization for the energy not supplied which is equal to $C(K - (We + P_g))$. Relation (2:4) is identical to the relation (2:3) when $W_e = 0$.

Let $\mathcal{M}$ be the expected profit function that is a function of both P_g and K variables:

$$\begin{aligned} \mathcal{M}(P_g, K) = &\ \mathbb{E}[\chi\{We + P_g \geq K\}\chi\{We > 0\}(\pi_e K - \pi_g P_g)] \\ &+ \mathbb{E}[\chi\{We + P_g < K\}\chi\{We > 0\}[\pi_e(We + P_g) - \chi_g P_g \\ &- C(K - (We + P_g))^{\alpha}] + \mathbb{E}[\chi\{We = 0\}[\pi_e P_g - \pi_g P_g - C(K - P_g)^{\alpha}]. \end{aligned} \tag{2.5}$$

Then, the optimization problem can be formalized as follows:

Maximize $\mathcal{M}(P_g, K)$

Subject to

$$\begin{aligned} h_1(P_g, K) &= \omega - P_g\pi_g \geq 0, \\ h_2(P_g) &= P_g \geq 0, \\ h_3(P_g, K) &= K - P_g \geq 0, \\ h_4(K) &= K \geq 0, \end{aligned} \tag{2.6}$$

where ω is the initial wealth. Noting that inequalities h_1 and h_3 assert that $P_g \leq \frac{\omega}{\pi_g}$ and $P_g \leq K$, it is possible to combine them in a single constraint introducing $A := \min\left\{\frac{\omega}{\pi_g}, K\right\}$. As a result, the optimization problem can be rewrite as:

Maximize $\mathcal{M}(P_g, K)$

Subject to

$$\begin{aligned} h_1(P_g, K) &= A - P_g \geq 0, \\ h_2(P_g) &= P_g \geq 0, \\ h_3(K) &= K \geq 0. \end{aligned} \tag{2.7}$$

The constraint h_1 has a double meaning depending on A whereby if $A = \frac{\omega}{\pi_g}$ the first bond says that the WPP can buy DES until his initial wealth is exhausted. Conversely when $A = K$ the interpretation of h_1 is that the quantity to put on the market has to be greater or at most equal to the quantity of DES purchased. The others two constraints, h_2 and h_3, guarantee that the quantity of DES bought and the quantity of energy offered on the market are greater or equal than zero, respectively.

To solve the optimization problem, it is necessary the introduction of the Lagrangian function of the problem:

$$\mathcal{L} = \mathcal{M}(P_g, K) + \lambda_1 h_1(P_g, K) + \lambda_2 h_2(P_g) + \lambda_3 h_3(K),$$

where λ_1, λ_2, λ_3 are the Lagrange's multipliers. The application of Kuhn-Tucker's theorem (Sundaram et al., 1996) gives a systems of equations based on A, where $A = \min\left\{\frac{\omega}{\pi_g}, K\right\}$.

Let discuss before the case when $K > \dfrac{\omega}{\pi_g}$, the other case, by similarity will shortly be discussed at the end of this section.

If $K > \dfrac{\omega}{\pi_g}$ then $h_1(P_g) = \dfrac{\omega}{\pi_g} - P_g$ and the application Kuhn-Tucker's theorem leads to the following system of equations:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_g} &= \left[C\alpha \int_0^{K-P_g} f_{we}(p)(K-(p+P_g))^{(\alpha-1)}\, dp + aC\alpha(K-P_g)^{\alpha-1} \right] \\
&\quad + \left[\int_0^{K-P_g} f_{we}(p)\mathbb{E}[\pi_e \,|\, We = p]dp + a\mathbb{E}[\pi_e \,|\, We = 0] \right] - \pi_g(1+a) \\
&\quad - \lambda_1 + \lambda_2 = 0 \\
\frac{\partial \mathcal{L}}{\partial K} &= \int_{K-P_g}^{+\infty} f_{we}(p)\mathbb{E}[\pi_e \,|\, We = p]dp + \lambda_3 - Ca\alpha[K-P_g]^{\alpha-1} \\
&\quad - \int_0^{K-P_g} f_{we}(p)C\alpha[k-p-P_g]^{\alpha-1} dp = 0,
\end{aligned} \tag{2.8}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \lambda_1} &= h_1(P_g) = \frac{\omega}{\pi_g} = P_g \geq 0, \\
\frac{\partial \mathcal{L}}{\partial \lambda_2} &= h_2(P_g) = P_g \geq 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda_3} &= h_3(P_g) = P_g \geq 0
\end{aligned}
$$

$$
\lambda_1 \geq 0,\ \lambda_2 \geq 0,\ \lambda_3 \geq 0,\ \lambda_1\left(\frac{\omega}{\pi_g} - P_g\right) = 0,\ \lambda_2 P_g = 0,\ \lambda_3 K = 0.
$$

The solution of system (2.8) passes through the consideration of six different cases:

- CASE I.1: $P_g = 0$, $\lambda_1 = 0$ and $\lambda_3 = 0$;
- CASE I.2: $P_g = 0$, $\lambda_1 = 0$ and $K = 0$;
- CASE I.3: $\lambda_2 = 0$, $P_g = A = \dfrac{\omega}{\pi_g}$ and $\lambda_3 = 0$;
- CASE I.4: $\lambda_1 = \lambda_2 = \lambda_3 = 0$;
- CASE I.5: $\lambda_1 = \lambda_2 = 0$ and $K = 0$;
- CASE I.6: $\lambda_2 = 0$, $P_g = A$ and $K = 0$.

Now let pay attention to three cases in which $K \neq 0$ that are the most interesting to be considered because if $K = 0$, the WPP does not participate in the production of energy neither with wind nor with DES.

2.1. Case I.1

$$P_g = 0,\ \lambda_1 = 0 \text{ and } \lambda_3 = 0$$

The first case, CASE I.1, is for $P_g = 0$, $\lambda_1 = 0$, $\lambda_3 = 0$. Thus, replacing these values to the first equation of system (2.8) it is obtained

$$-\pi_g(1+\alpha) + \int_0^K f_{we}(p)\mathbb{E}[\pi_e | We = p]dp + a\mathbb{E}[\pi_e | We = 0]$$
$$+aC\alpha(K)^{\alpha-1} + \int_0^K f_{we}(p)C\alpha(K-p)^{\alpha-1}dp = 0. \tag{2.9}$$

From the second equation it is recovered

$$Ca\alpha(K)^{\alpha-1} = \int_K^{+\infty} f_{we}(p)\mathbb{E}[\pi_e | We = p]dp - \int_0^K f_{we}(p)C\alpha(K-p)^{\alpha-1}dp, \tag{2.10}$$

that can be substituted in formula (2.9) and, after remarking that $\lambda_2 > 0$, it is achieved

$$\pi_g > \frac{\int_0^{+\infty} f_{we}(p)\mathbb{E}[\pi_e | We = p]dp + a\mathbb{E}[\pi_e | We = 0]}{1+a} = \frac{E[\pi_e]}{1+a}.$$

In this situation it is not optimal to use gas (in fact $P_g = 0$) if the price of gas (π_g) is greater than the expected price of electricity divided by $(1 + a)$. Accordingly, the optimal solution suggests using only wind power so that $P_g = 0$. The optimal quantity of energy to plan to produce in the next period can be found numerically using the second equation of system (2.8), which, after the substitution $P_g = 0$ can be rewritten as follows,

$$\frac{\partial}{\partial K}\left[K\int_K^{+\infty} f_{we}(p)\mathbb{E}[\pi_e | We = p]dp\right] + Kf_{we}(K)\mathbb{E}[\pi_e | We = K]$$
$$-\frac{\partial}{\partial K}\left[\int_0^K f_{we}(p)C[K-p)^{\alpha}dp\right] - \frac{\partial}{\partial K}[aCK]^{\alpha} = 0. \tag{2.11}$$

It is possible to note that the quantity

$$-\frac{\partial}{\partial K}\left[\int_0^K f_{we}(p)C[K-p)^{\alpha}dp\right] - \frac{\partial}{\partial K}[aCK]^{\alpha}, \tag{2.12}$$

is the total marginal penalization considering production of energy both with wind and without wind. Whereas the quantity

$$\frac{\partial}{\partial K}\left[K\int_0^{+\infty} f_{we}(p)\mathbb{E}[\pi_e | We = p]dp\right], \tag{2.13}$$

is the marginal revenue in case of overproduction.

2.2. Case I.3

$$\lambda_2 = 0,\ P_g = A = \frac{\omega}{\pi_g} \text{ and } \lambda_3 = 0$$

Another solution of system (2.8) arises when $\lambda_2 = 0$, $P_g = A = \frac{\omega}{\pi_g}$, $\lambda_3 = 0$. Applying these conditions to the first and second equations of the system (2.8) and using similar arguments as those exposed in the former case it is obtained:

$$\pi_g < \frac{E[\pi_e]}{1+a}.$$

In this case it is optimal produce the maximum with gas because the price of gas is lower than $\frac{E[\pi_e]}{1+a}$. Again K is determined numerically compared to the quantity $(K - P_g)$, where $P_g = \frac{\omega}{\pi_g}$, by solving with respect to K the following equation:

$$C a\alpha\left(K - \frac{\omega}{\pi_g}\right)^{\alpha-1} = \int_{K-\frac{\omega}{\pi_g}}^{+\infty} f_{we}(p)\mathbb{E}[\pi_3 | We = p]dp - \int_0^{K-\frac{\omega}{\pi_g}} f_{we}(p)C\alpha\left(K - p - \frac{\omega}{\pi_g}\right)^{\alpha-1} dp \quad (2.14)$$

2.3. Case I.4

$$\lambda_1 = \lambda_2 = \lambda_3 = 0$$

The last non trivial solution for the system (2.8) is for $\lambda_1 = \lambda_2 = \lambda_3 = 0$. First, it is necessary to substituite $\lambda_1 = \lambda_2 = \lambda_3 = 0$ into system (2.8). Then, from the second equation of the system, it is obtained

$$C a\alpha(K - P_g)^{\alpha-1} = \int_{K-P_g}^{+\infty} f_{we}(p)\mathbb{E}[\pi_e | We = p]dp - \int_0^{K-P_g} f_{we}(p)C\alpha(K - p - P_g)^{\alpha-1}dp, \quad (2.15)$$

that can be substituted into the first equation of system (2.8). Simple algebraic calculations give

$$\pi_g = \frac{\int_0^{+\infty} f_{we}(p)\mathbb{E}[\pi_e | We = p]dp + a\mathbb{E}[\pi_e | We = 0]}{1+a},$$

In other words

$$\pi_g = \frac{E[\pi_e]}{1+a}.$$

Equation (2.15) shows a dependence only on the difference between K and P_g thus it is convenient to introduce a new variable $\gamma = K - P_g$. In this way, it is possible to solve numerically equation (2.15) with respect to γ and get an optimal value γ. Consequently infinite combinations of P_g and K that gives γ^*, *i.e.* $K = P_g + \gamma^*$, are available. This reveals that, whenever $\gamma_1 = \gamma_2 = \gamma_3 = 0$, it is possible to determine P_g according to previous results (D'Amico et al., 2021) and then, compute K such that $K = P_g + \gamma^*$. In Figure 2, it is provided a graphical illustration to better understand this interpretation. The blue line shows all of the optimal combinations of K and P_g while points A and B describe two different scenarios. In detail, point B describes, for a fixed level of P_g, the situation in which the energy offered on the market is too low and it is appropriate to increase this offer up to the blue line. Viceversa point A displays the situation in which the quantity of energy provided is too high and it is beneficial to decrease it until the optimal line because it is expected a lower total power production that, with an excessive offer, will generate a penalty.

Now let briefly discuss the scenario in which $K < \frac{\omega}{\pi_g}$. The relative system of equations is similar to system (2.8). The only difference is in the second equation in which it is necessary to add the term λ_1, obtaining:

$$\frac{\partial \mathcal{L}}{\partial K} = \int_{K-P_g}^{+\infty} f_{we}(p)\mathbb{E}[\pi_e \,|\, We = p]dp + \lambda_3 - Ca\alpha[K - P_g]^{\alpha-1}$$
$$-\int_0^{K-P_g} f_{we}(p)C\alpha[K - p - P_g]^{\alpha-1}dp + \lambda_1 = 0.$$

Also in this system of equations it is appropriate to consider only the cases in which $K \neq 0$. It is obtained that the case in which P_g, λ_1 and λ_3 are equal zero, and the case in which λ_1, λ_2 and λ_3 are equal zero, lead to the same result and interpretation of CASE I.1 and CASE I.4 respectively. It is achieved a different result for the case in which λ_2, λ_3 are equal zero and $P_g = K$ compared to the case λ_2, λ_3 equal zero and $P_g = \frac{\omega}{\pi_g}$ (CASE I.3). In particular it is attained that, if $\pi_g < \frac{\alpha\mathbb{E}[\pi_e | We = 0]}{1+a}$, the WPP will use only gas $\forall\, K < \frac{\omega}{\pi_g}$.

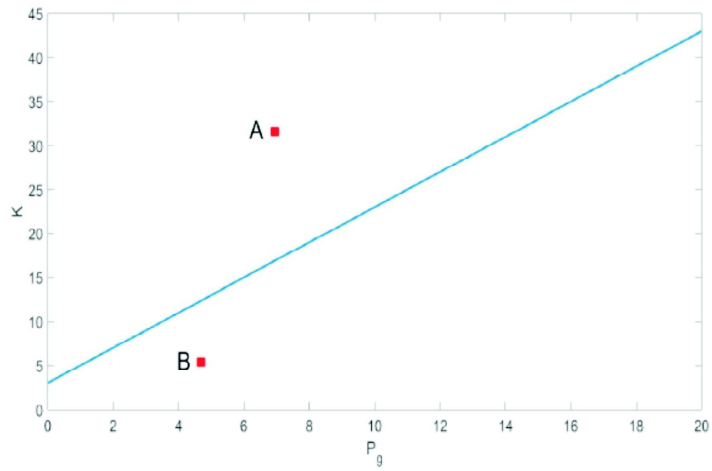

Figure 1: Graphic illustration of the solution obtained for $\lambda_1 = \lambda_2 = \lambda_3 = 0$ with P_g and K as control variables

3. EMPIRICAL FEATURES OF THE MODEL

The advanced model requires all the involved quantities to be set operationally. To this purpose, it is applied to a hypothetical wind farm of 48 MW rated power which consists of 24 independent wind turbines. Consequently, the total wind farm production is given by multiplying the number of turbines with the unitary wind power production. The choice of independent wind turbines is carried out only to simplify the investigation. In fact, in a real scenario, many aspects can induce correlation between turbines such as shear effects and ground geomorphological structures.

Specifically, wind data have been downloaded from NASA's MERRA-2 database (https://gmao.gsfc.nasa.gov/reanalysis/MERRA-2) and then, converted into wind energy using a power curve.

This trasformation is carried out assuming for each turbine:

- Geographical coordinates- 39:5 N (latitude) and 8:75 E (longitude),
- Hub height-95 m,
- Rated power- 2 MW,
- Cut-in wind speed-13 m/s,
- Cut-out wind speed-25 m/s,
- Rated wind speed-13 m/s,

Electricity prices data have been downloaded from Borsa Elettrica Italiana (https://www.mercatoelettrico.com). Both series have hourly resolution and refer to the period 2008-2018.

The unit of energy produced is MWh while the unit of electricity prices is €/MWh.

According to the literature (D'Amico et al., 2021), it is assumed that the wind power production has a mixed discrete-continuous distribution with the following cdf:

$$F_{we}(p) = \begin{cases} 0 & \text{if } p < 0 \\ a + (1+a)\left(1 - e^{-\left(\frac{p}{\lambda}\right)^{\gamma}}\right) & \text{if } p \geq 0,\, a \in [0,1]. \end{cases}$$

where $F_{we}(p)$ has a Weibull distribution for his continuous part, *i.e.* in the interval (0, +∞), and a is a point mass at zero. As for the electricity price, it is used a Lognormal distribution for computational reasons (D'Amico et al., 2021). By Matlab software, it is possible to estimate all the parameters of the selected distributions. In detail, the parameters λ and γ of the Weibull distribution are computed using the function 'fitdist' and their estimated values are equal to 11.91 and 0.800 with corresponding 95% confidence interval [11:79, 12:03] and [0:795, 0:805]. The point mass at zero a is the parameter of a Bernoulli variable thus, it is determined as the ratio between the number of null wind power production events and the total number of observations. Its value is equal to 0.26 and it is within the corresponding 95%-confidence interval of [0:24, 0:28]. Also the parameters and of the Lognormal distribution are estimated using the function 'fitdist' and are equal to 4.034 e/MWh and 0.719 e/ MWh with respective 95%-confidence intervals of [4.029, 4.038] and [0.715, 0.723].

Figure 2 shows a comparison between the empirical cumulative distribution of positive wind energy production data and the theoretical Weibull distribution. Similarly, Figure 3 compares the empirical cumulative distribution of electricity prices data and the theoretical Lognormal distribution.

In general, it is difficult to fix a value for the parameters α and C because they depend on the specific signed contract. For example, it could be reasonable to set C equal to the average electricity price which, in this application, is equal to 64.563 €/MWh. Regarding the parameter α, it is required a more careful analysis as it plays a fundamental role in the penalty scheme. In fact, it should be avoided values of α in the interval (0, 1) because they penalize marginally the smaller deviations from the target more than

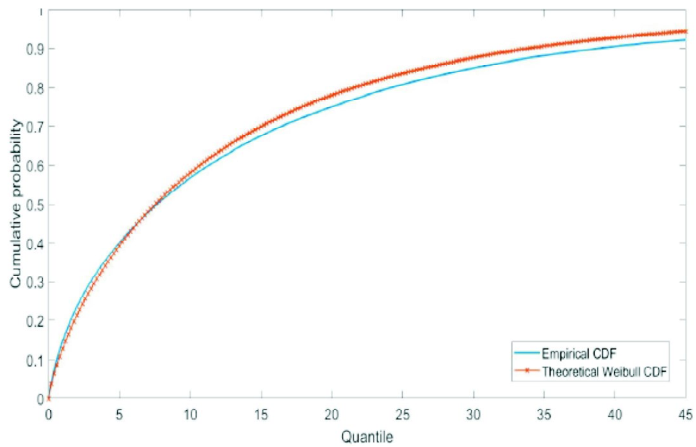

Figure 2: Empirical and theoretical CDF of wind energy production

the large deviations. A good choice could be the use of values greater than 1 that allow to adequately penalize greater deviations from the target, more than smaller ones.

Finally, the correlation ρ between the electricity prices and wind power production is negative and equal to –0.0544. This result is in line with the selection of the FGM copula as it is able to manage only correlations in the range $\left[-\frac{1}{3}, \frac{1}{3}\right]$. Consequently, the parameter is equal –0.1631 as $\theta = \rho \times 3$ (Durante and Sempi, 2016)-(Bekrizadeh, Parham and Zadkarmi, 2012).

4. CONCLUSION

The methodology proposed in this paper, which is an improvement of that already presented in literature (D'Amico et al., 2021), develops optimal strategies for wind energy management. By solving a non-linear optimization problem with a budget constrain, the optimal quantity of wind energy to be produced using dispatchable energy sources P_g and the optimal quantity of energy K to be offered on the market are detected.

The model considers P_g and K as control variables and determines the optimal policies according to the initial wealth endowment, the unpredictable production of wind energy, the electricity prices, the penalty system, and their inter-dependencies.

All the quantities involved in the model have been commented and concretely identified through an empirical investigation conducted on a supposed wind farm.

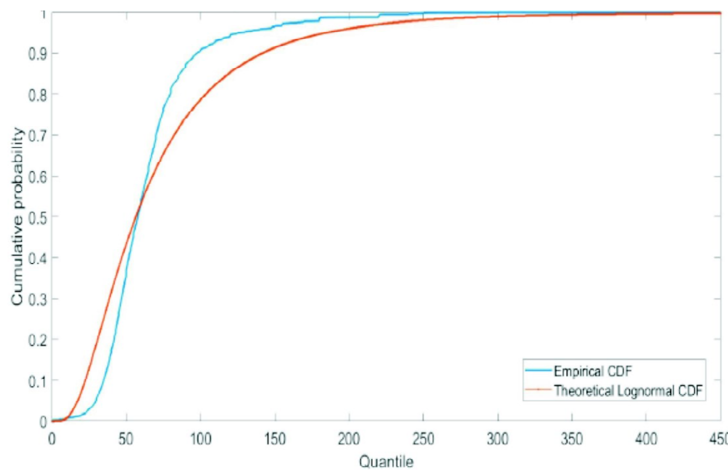

Figure 3: Empirical and theoretical CDF of electricity prices

Obviously the model leaves space for new possible extensions. Firstly, it could be expanding to a multi-period scenario in which optimal policies are repeatedly determined. Secondly, the possibility of borrowing money to grow the initial endowment could be introduced. Thirdly, the distributions belonging to the Extreme Value Theory class could be used to model extreme variations in both electricity prices and the speed of energy.

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